

A Braneworld Universe From Colliding Bubbles

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Abstract

Much work has been devoted to the phenomenology and cosmology of the so-called braneworld universe, where the $(3+1)$ -dimensional universe familiar to us lies on a brane surrounded by a $(4+1)$ -dimensional bulk spacetime that is essentially empty except for a negative cosmological constant and the various modes associated with gravity. For such a braneworld cosmology, the difficulty of justifying some set of preferred initial conditions inevitably arises. The various proposals for inflation restricted to the brane only partially explain the homogeneity and isotropy of the resulting braneworld universe because the three-dimensional homogeneity and isotropy of the bulk must be assumed *a priori*. In this paper we propose a mechanism by which a brane surrounded by AdS space arises naturally in such a way that the homogeneity and isotropy of both the brane and the bulk are guaranteed. We postulate an initial false vacuum phase of $(4+1)$ -dimensional Minkowski or de Sitter space subsequently decaying to a true vacuum of anti-de Sitter space, assumed discretely degenerate. This decay takes place through bubble nucleation. When two bubbles of the true AdS vacuum eventually collide, because of the degeneracy of the true AdS vacuum, a brane (or domain wall) inevitably forms separating the two AdS phases. It is on this brane that we live. The $SO(3,1)$ symmetry of the collision geometry ensures the three-dimensional spatial homogeneity and isotropy of the universe on the brane as well as of the bulk. In the semi-classical ($\hbar \rightarrow 0$) limit, this $SO(3,1)$ symmetry is exact. We sketch how the leading quantum corrections translate into cosmological perturbations.

I. INTRODUCTION

We propose a cosmogony based on collisions of true anti-de Sitter (AdS) vacuum bubbles in $(4+1)$ dimensions expanding at nearly the speed of light within a surrounding $(4+1)$ -dimensional de Sitter (dS) or Minkowski (M) space false vacuum. The bubble collisions produce a braneworld universe very similar to the cosmogony with a $(3+1)$ -dimensional,

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positive-tension brane surrounded by $(4 + 1)$ -dimensional AdS space proposed by Randall and Sundrum [5,10].

Initially a $(4 + 1)$ -dimensional spacetime consisting of either de Sitter space or Minkowski space is supposed. In the former case an initial epoch of $(4 + 1)$ -dimensional ‘old’ inflation [2] ensures a very nearly $SO(5, 1)$ symmetric state prior to bubble nucleation, regardless of whatever departures from de Sitter space may have initially existed. The homogeneity and isotropy of the resulting $(3 + 1)$ -dimensional braneworld universe is thus assured, as we shall explain in more detail. In the latter case, a metastable $(4 + 1)$ -dimensional Minkowski state vacuum must be postulated at the outset; however, it is not at all implausible that some as yet unknown theory of the initial conditions of the universe prefers empty Minkowski space.

The false de Sitter or Minkowski vacuum decays through the nucleation by quantum tunneling of bubbles filled with the lower energy true AdS vacuum [6,7]. The bubble wall separating the two phases may take the form of either a brane or an accelerating domain wall. We postulate that the AdS vacuum is discretely degenerate, so that the energy from the collision of two bubbles is not entirely transformed into energy dispersed into the $(4 + 1)$ dimensions. In the case of a degenerate AdS vacuum, when the two colliding bubbles contain differing AdS phases, after the collision at least part of the energy is transferred to a brane (or domain wall) that must mediate between the two phases. This is energy that remains localized in the fifth dimension. In this paper we shall call this brane (or domain wall) our *local brane* because this is where the $(3 + 1)$ -dimensional universe familiar to us resides.

To the extent that our universe has a violent beginning resulting from the collision of branes, the scenario presented here has much in common with the ekpyrotic universe recently proposed by Khoury et al. [1]; however, the physics by which preferred initial conditions are determined is quite different. Perkins [13] considered a braneworld scenario in which our universe is situated on a bubble wall. However, in his scenario bubble collisions are catastrophic. The dynamics of bubble collisions have been studied by Guth and Weinberg [4], Hawking, Moss, and Stewart [12], and Chao [11].

Before embarking on a detailed description of the colliding bubble scenario, we first highlight some of the problems arising from the assumption of a bulk with a negative cosmological constant. These difficulties, which render many braneworld cosmogonies problematic, are avoided in the scenario proposed here because of the presence of a prior epoch of de Sitter or Minkowski space. Most braneworld models, including those with inflation on the brane, are plagued by the same horizon and smoothness problems present in non-inflationary cosmogonies but in $(4 + 1)$ rather than $(3 + 1)$ dimensions. The persistence of very near spatial homogeneity and isotropy on the brane requires that the bulk at the outset be very nearly three-dimensionally homogeneous and isotropic. Otherwise, through gravity an inhomogeneous bulk inevitably induces inhomogeneities on the brane. A successful braneworld cosmology must therefore explain why the bulk was very nearly homogeneous and isotropic at the beginning. A mechanism that merely smooths an initially inhomogeneous brane embedded in pristine AdS space, such as brane inflation, does not suffice because the necessary bulk homogeneity and isotropy must be put in by hand.

Anti de Sitter space, or more broadly any spacetime with the stress-energy of a negative cosmological constant, lacks the ability to erase small initial perturbations from homogeneity and isotropy. For the case of a positive cosmological constant departures from homogeneity and isotropy rapidly disappear as the universe expands. This is what provides the magic of

inflation, by which perturbations are rapidly stretched to scales too large to be observable so that after a rather modest amount of expansion one is left with essentially stretched vacuum. One could perversely attempt to postulate some sort of fractal state for which no amount of inflationary expansion would yield a homogeneous and isotropic state on the scales of interest, but such an initial state would entail an infinite energy density and thus can be excluded, even under the most generous restrictions on admissible initial states.

The differing evolutions of dS and AdS space are readily illustrated by considering the family of timelike geodesics emanating from some arbitrary point in the spacetime, as indicated in Fig. 1. One might for example interpret these geodesics as the worldlines of the shrapnel from an exploding bomb! In both cases the trajectories initially diverge according to their relative velocities, just as in a Milne universe (which is but another coordinatization of flat Minkowski space). However, after a proper time comparable to the curvature radius, the trajectories in AdS start to converge, eventually refocusing to a point (where the bomb momentarily re-assembles itself!), and this sequence of divergence and reconvergence repeats itself *ad infinitum*. In de Sitter space, however, the nonvanishing spacetime curvature has exactly the opposite effect. Rather than re-converging the initial linear divergence of the trajectories accelerates, so that eventually the pieces of shrapnel lose causal contact with each other. Exponentially inflating spacetime inserts itself between the fragments. In summary, de Sitter space loses its “hair”, while anti de-Sitter space most definitely does not.

In addition to the persistence of the irregularities in the manner just described, anti-de Sitter space is plagued with a bizarre causal structure. As indicated in Fig. 1(a), maximally extended AdS space is bounded by timelike boundaries at infinity from which and to which information flows. It does not make sense to postulate eternal AdS without some theory of appropriate boundary conditions on these edges or on the Cauchy horizons that result when one attempts to limit consideration to a subspace of the maximally extended spacetime. In the original Randall-Sundrum proposal (whose causal structure is indicated in Fig. 2), the usual Randall-Sundrum coordinates

$$ds^2 = dy^2 + \exp[2y] \left[-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] \quad (1)$$

cover only a minute portion of maximally extended AdS space. The coordinate patch covered by (1) forms a globally hyperbolic subspace of maximally extended anti de Sitter space—that is, initial data on a slice of constant cosmic time in the Randall-Sundrum coordinates is not constrained by any consistency conditions and completely suffices to determine the fields in the triangular region covered by these coordinates. But the lower light-like boundary constitutes a Cauchy horizon, and one may legitimately inquire, what principle determines the initial conditions on this boundary? And if they are trivial, as is often assumed, why is this so? Although Fig. 2 represents a static Randall-Sundrum universe, the lower Cauchy horizon persists in Randall-Sundrum cosmological models with an expanding universe.

In the proposal presented here AdS bubbles arise through the decay of a false de Sitter space or Minkowski space false vacuum. The AdS space that emanates inside the bubble is produced in a precise and predictable way, with quantum fluctuations that are predictable and calculable. The problems described above are avoided. In the next section, we describe the geometry and dynamics of the production and collisions of AdS bubbles, explaining why in the semiclassical $\hbar \rightarrow 0$ limit the resulting brane universe is homogeneous and isotropic. In section III we turn to the leading quantum corrections to this picture, presenting a

simplified calculation of the quantum fluctuations, which in our universe translate into a spectrum of Gaussian linearized cosmological perturbations. In the final section we present some concluding remarks.

II. ADS FROM COLLIDING BUBBLES

The possibility has been previously advanced that the true vacuum might not be what we commonly perceive as the true vacuum. That is, rather than being either empty Minkowski space or de Sitter space with a remarkably small positive cosmological constant, the true vacuum might take the form of some lower energy state with a negative cosmological constant. If this were true, we would live in a metastable false vacuum state susceptible to decay to the true vacuum through bubble nucleation. Phenomenologically, given the observed persistence of our universe, an approximate upper bound on the rate Γ at which bubbles of true AdS vacuum spontaneously nucleate can be established, but it is not possible to reject this possibility altogether.

A manifestly covariant description of the dynamics of false vacuum was first given by Sidney Coleman, first ignoring gravity [6] and then extended to include the gravitational corrections in work with F. de Luccia [7]. Important prior work is contained in [8]. Below we summarize the principal results of these papers to the extent that they are needed here and refer the reader to the original papers for a more detailed and rigorous discussion.

False vacuum decay takes place at zero temperature, or said another way, from an initial state no preferred time direction. The consequences of the lack of a preferred time direction are profound. They render false vacuum decay qualitatively different from the more familiar thermal tunnelling, which enjoys considerably less symmetry due to the fact that a thermal state has a preferred time direction. For false vacuum decay in $(d + 1)$ dimensions, the resulting classical expanding bubble solution possesses an $SO(d, 1)$ symmetry. The symmetry group (which in the case of de Sitter space would be $SO(d + 1, 1)$) is broken to $SO(d, 1)$, the subgroup of transformations that leaves invariant a spacetime point known as the *nucleation center*.

For a spacetime with two bubbles, the resulting symmetry is further reduced, but considerable residual symmetry remains. Suppose that two bubbles nucleate at spacelike separated nucleation centers N_1 and N_2 . (The separation must be spacelike, for otherwise one bubble would nucleate within the other.) For two bubbles the solution remains symmetric under the subgroup of those transformations that leave invariant the line (or spacelike geodesic) passing through N_1 and N_2 . For a pair of colliding bubbles nucleating in $(4+1)$ -dimensional dS space, the $SO(5, 1)$ symmetry breaks to $SO(3, 1)$. This residual symmetry has the following consequences. First, one may always choose a coordinate system in which the two bubbles nucleate at the same time. Hence, unlike for thermal tunneling, here it does not make sense to ask which bubble was the bigger one. Moreover, once a coordinate choice is made in which the bubbles nucleate simultaneously, substantial residual symmetry still remains. While in a particular coordinate system the bubbles first collide at a given spacetime point P , for any other point P' belonging to the locus of points where the bubbles collide, a coordinate transformation exists such that the bubbles first collide at P' . It is this symmetry mapping P into P' that is responsible for the three-dimensional spatial homogeneity and isotropy of the universe on the local brane.

We now turn to a more detailed consideration of what happens during the bubble collision. For vacuum decay with a single scalar field where the AdS vacuum is nondegenerate, the energy of the colliding bubble walls, absent some good reason to the contrary, dissipates in the fifth dimension (the direction parallel to the line connecting the two nucleation centers) but in an $SO(3, 1)$ symmetric way, much as in the initial stages of thermalization first envisaged for ‘old’ [2] or ‘extended’ [3] inflation. However, if the AdS vacuum is finitely degenerate (in the simplest case there are two such AdS vacua related by a Z_2 symmetry), topology demands that a domain wall form after the bubble collision to separate the two distinct AdS domains when the colliding bubbles contain differing AdS phases. While energy that disperses in the fifth dimension could as well be produced in the collision, topology requires that a domain wall form to mediate between the two AdS states. This wall, which we call our *local brane* (on which we live) is at rest in the center of mass frame of the colliding bubble. Of the kinetic energy left over after this domain wall has been formed a part is expected to stick to the brane (and be confined to it, as is typically assumed in the Randall-Sundrum scenario) and another part is expected to disperse into the bulk. The energy dispersed into the bulk, however, is $SO(3, 1)$ symmetric, and therefore does not induce any irregularities on the brane. Moreover, this energy does not fall back onto the brane, because when the gravitation of the brane is taken into account, the brane accelerates away from this symmetric dispersive debris.

In Fig. 4 we show schematically, first a 2+1 dimensional representation of the colliding bubble geometry in the left panel and then in the right panel a cut-away of the surface of equal proper distance from the two nucleation centers. The curve labeled C indicates the line along which the two bubbles collide. In the section on the right, four hyperbolic coordinate patches are generated by the $SO(4, 1)$ symmetry. Points along the solid curves are rendered equivalent by this symmetry. These are lines of constant cosmic temperature on our local brane, which cools as the universe expands. In the full 3+1 dimensional case, these curves are three-dimensional spacelike hyperboloids of constant negative spatial curvature.

It has been suggested by Coleman that isolated AdS bubbles generically collapse into black holes because of the $SO(4, 1)$ symmetry of the perfect classical expanding bubble solution. The argument, which is closely related to the perfect refocusing of timelike geodesics emanating from a point described above, is as follows. The universe inside an AdS bubble is a hyperbolic universe that recollapses after a finite amount of time. If the background stress-energy inside the bubble were that of a perfect negative cosmological constant, this would pose no problem. The resulting ‘Big Crunch’ would be nothing but a coordinate artifact, as indicated in Fig. 5. However, if the scalar field undergoing tunneling has not reached the true vacuum by the light cone L (which it never does), a singularity in the evolution of the scalar field on the light cone L' generally results. In both cases the behavior of the scalar field near the lightcones is described by a second-order, Bessel-like ordinary differential equation having one regular and one singular solution. On L it is clearly correct to choose the regular solution. This is the initial condition resulting from the Euclidean instanton. But unless the potential is extremely finely tuned, upon propagating to L' , at least a small admixture of the divergent, irregular solution will be chosen causing the scalar field kinetic energy to diverge. In the case of colliding bubbles, however, the underlying symmetry that led to the divergence is broken because the collision generically sends out a wave that spoils the finely tuned convergence of the scalar field that led to black hole

formation. Thus the AdS space inside the bubbles is allowed to persist.

To simplify the analysis, we idealize the bubble walls as infinitely thin and assume that upon colliding the bubbles transfer all their available energy onto an infinitely thin brane, with all excess energy converted into radiation and matter confined to the brane. The collision geometry is indicated in Fig. 6. The subsequent evolution of the brane depends on the equation of state on our brane, which we take to be arbitrary, since the considerations presented in this paper do not depend on the details of this.

Since bubbles nucleate stochastically, at a rate Γ with the dimension of inverse volume inverse time, the proper distance between nucleation centers is a random variable. Consequently, the spatial curvature of the resulting intermediate brane universe varies between bubble pairs. In this scenario it is essential that bubble collisions are rare. A collision with a third bubble would be catastrophic; a wave of energy would move toward us at very nearly the speed of light striking us with essentially no warning. That this has not yet happened is a trivial application of the anthropic principle.

III. QUANTUM CORRECTIONS: GENERATION OF GAUSSIAN COSMOLOGICAL PERTURBATIONS

In the previous section we demonstrated how a homogeneous and isotropic universe can arise from the collision of two expanding AdS bubbles. We employed the semiclassical ($\hbar \rightarrow 0$) limit in which prior to colliding each bubble possesses an exact $SO(4, 1)$ symmetry about its nucleation center, because in the semiclassical limit fluctuations about the configuration of least Euclidean action describing the bubble nucleation process are suppressed as well as the quantum fluctuations of the wall and of the surrounding fields afterward. In this limit one obtains an absolutely homogeneous and isotropic universe, quite unlike the one that we observe. Quantum corrections, however, alter this picture. The leading order corrections in \hbar yield a calculable spectrum of linearized Gaussian fluctuations. These are the usual Gaussian cosmological perturbations.

For calculating the cosmological perturbations, the Bunch-Davies vacuum of de Sitter space (or the Minkowski space vacuum for the case of bubbles nucleating in Minkowski space) define a natural set of initial conditions. The Bunch-Davies vacuum is an attractor, so an initial state deviating from this state evolves to become successively well approximated by the Bunch-Davies vacuum. A full calculation of the perturbations is postponed until a later paper [14]. Here we limit ourselves to a simplified description ignoring gravitational back reaction and assuming infinitely thin bubbles to illustrate the underlying physical processes.

The quantum state for the fluctuations of a thin wall bubble about the perfect $SO(4, 1)$ expanding bubble solution for a bubble arising from false vacuum decay was first elucidated by J. Garriga and A. Vilenkin [9]. In the thin wall approximation, with the gravitational back reaction of the perturbations ignored, the only available degree of freedom consists of normal displacements of the bubble wall, which may be described as a scalar field on the wall. We consider the perfect $SO(4, 1)$ symmetric expanding bubble (which has the geometry of de Sitter space). Displacements along the outward normal are described by a free scalar field with mass ($M = -4H^2$). The quantum state of this field must obey the same $SO(4, 1)$ symmetry as the classical expanding bubble solution. One might at first sight admit the possibility that bubble nucleation could somehow spontaneously single out a preferred time

direction. That this is not possible can be demonstrated by contradiction. Suppose that such a choice of preferred time direction were in fact made. Then all such choices must be equally weighted, according to a Lorentz invariant measure. The calculation of the vacuum decay rate would contain a factor consisting of an integration over the infinite hyperbolic domain of all such possible choices, thus implying an infinite false vacuum decay rate, which is clearly absurd. The $SO(4,1)$ invariance of the quantum state of the fluctuations suffices to completely fix this state. It is described by the Bunch-Davies vacuum on the de Sitter space of the expanding bubble wall.

Let χ_1 and χ_2 be the scalar fields just described for the two colliding bubbles, using the sign convention that χ is positive for outward displacements. To analyse how these displacements translate into perturbations in the brane arising from the bubble collision, it is convenient to consider the linear combinations

$$\begin{aligned}\chi_+ &= (\chi_1 + \chi_2)/\sqrt{2}, \\ \chi_- &= (\chi_1 - \chi_2)/\sqrt{2}\end{aligned}\tag{2}$$

at the instant of collision. The mode χ_+ temporally advances (or retards) the surface on which the bubbles collide leading to under and overdensities. The hyperboloids of constant cosmic temperature are thus warped. This mode translates into scalar density perturbations of the cosmology on the local brane. The mode χ_- , on the other hand, displaces the points of collision in the normal direction—that is, spatially toward the one or the other bubble.

Although the geometry of the background solution is Z_2 symmetric as in the Randall-Sundrum scenario, the Z_2 symmetry here is qualitatively different from the orbifold Z_2 symmetry postulated in the Randall-Sundrum proposal. In our proposal, both Z_2 even and Z_2 odd perturbations are allowed because the degrees of freedom on one side of the brane do not coincide with those on the other side. In the Randall-Sundrum scenario with a single brane there is no bending mode because the relevant degrees of freedom have been decreed not to exist through the orbifold construction. In our case, this mode does in fact exist. The extrinsic curvature (relative to the outward normal) on the two sides need not coincide because twice as many degrees of freedom are present.

IV. CONCLUDING REMARKS

We have demonstrated how the collision of two bubbles filled with AdS space expanding in de Sitter space or Minkowski space can give birth to a braneworld cosmology surrounded by infinite anti de Sitter space, very similar to the single-brane Randall-Sundrum model. In this colliding bubble scenario well-defined initial conditions naturally arise. The smoothness and horizon problems in $(4 + 1)$ dimensions are absent in this scenario. Although the considerations in this paper apply equally well regardless of the equation of state on the local brane produced after the bubble collision, the fact that inflation on the resulting $(3+1)$ dimension spatially hyperbolic universe can altogether be avoided is intriguing. If sufficient energy is deposited on this brane after a sufficiently long expansion time, Ω today can be very close to one.

We now consider some orders of magnitude. In the Randall-Sundrum scenario (just as in compact five-dimensional Kaluza-Klein models), an effective four-dimensional Planck mass

M_4 large compared to the five-dimensional Planck mass M_5 may be obtained by making the size of the extra dimension ℓ large. Here we set $\hbar = c = 1$. In the RS case ℓ is the curvature radius of the AdS bulk. Since $M_4^2 = M_5^3 \ell$, $M_4 = M_5(M_5 \ell)^{1/2}$.

The five-dimensional Einstein equation and Israel matching condition give $\Lambda = M_5^3 \ell^{-2}$ and $\sigma = M_5^3 \ell^{-1}$, respectively. The tension of the wall separating the AdS from the dS phase and that of the local brane in general differ, but for the order-of-magnitude analysis here we take them to coincide. It follows that the approximate size of the critical bubble is $r \approx \sigma/\Lambda \approx \ell$. The vacuum decay rate is proportional to $\Gamma = \exp[-S_E]$ where $S_E \approx \sigma r^4 \approx (M_5 \ell)^3 = (M_4 \ell)^2$. An extra dimension large compared to the Planck scales makes the dimensionless Euclidean action large, leading to an exponentially small bubble nucleation rate. Therefore, a very substantial amount of expansion takes place before bubble pairs collide, and three bubble collisions are rare. The perturbations are of order $1/\sqrt{S_E}$, which can easily be adjusted to match the observed primordial anisotropies.

We now consider the spatial curvature of the universe on the local brane. The energy density on the brane produced at the bubble collision is approximately $E_C = (R/r)\sigma$ where $\sigma \approx M_5^4 (M_5 \ell)^{-1}$ and R is the distance between the nucleation centers of the two bubbles. As long as $(R/r) \lesssim (\ell/\ell_5)$ this energy density is sub-Planckian from the five-dimensional point of view.¹ At the collision $(1 - \Omega_c) \approx (\ell/R)^3$, which is exponentially small. The factor e^{-S_E} provides a natural mechanism to adjust Ω_c so close to one that Ω today remains very close to one without resort to unnatural fine tuning.

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¹The case of supra-Planckian energy densities immediately after the brane collision, however, need not necessarily be discarded, because the analysis of what happens afterward depends little on the details on how the universe cools down. One might regard a brief supra-Planckian epoch after the collision as a sort of black box, much as re-heating at the end of inflation is commonly regarded. One is able to compute the perturbations from inflation with complete confidence despite our almost total ignorance of how re-heating occurs.

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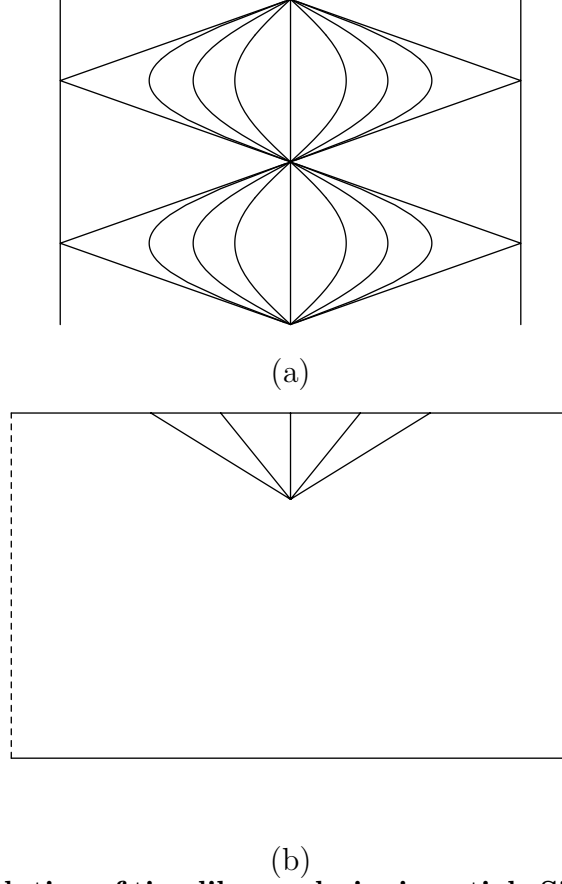


FIG. 1. **Differing evolution of timelike geodesics in anti de Sitter and de Sitter space.** The left panel (a) shows a conformal diagram for anti-de Sitter space, which has the form of an infinite vertical strip of finite thickness. The horizontal and vertical directions indicate space and time, respectively, and null geodesics travel obliquely at 45 degrees. The right panel (b) shows the conformal diagram of de Sitter space which has the form of a cylinder of finite height. The dashed vertical boundaries are identified. In both panels the forward timelike geodesics of a spacetime point P are indicated, as well as the asymptotic light cones forming the boundary of the causal future of P . In anti-de Sitter space the timelike geodesics periodically refocus ad infinitum. By contrast, in de Sitter space the geodesics diverge, eventually losing causal contact with each other.

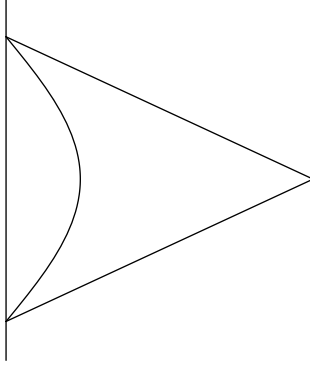


FIG. 2. **Causal structure of the single-brane Randall-Sundrum braneworld space-time.** The surfaces of constant time in the Randall-Sundrum coordinates are generated by the family of all spacelike geodesics emanating from a certain point on the conformal boundary. The worldlines of constant transverse coordinate (i.e., the “fifth dimension” of the Randall-Sundrum scenario) represent uniformly accelerating observers all with the same uniform acceleration away from this point. The Cauchy horizons H_- and H_+ are the past and future boundaries of the region covered by these coordinates.

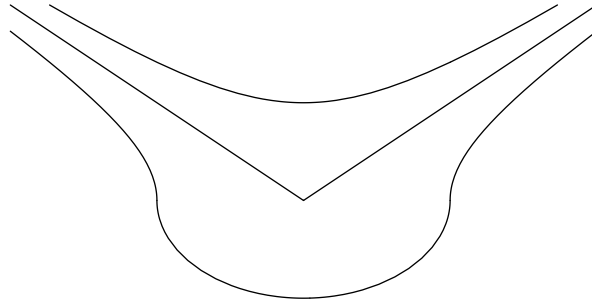
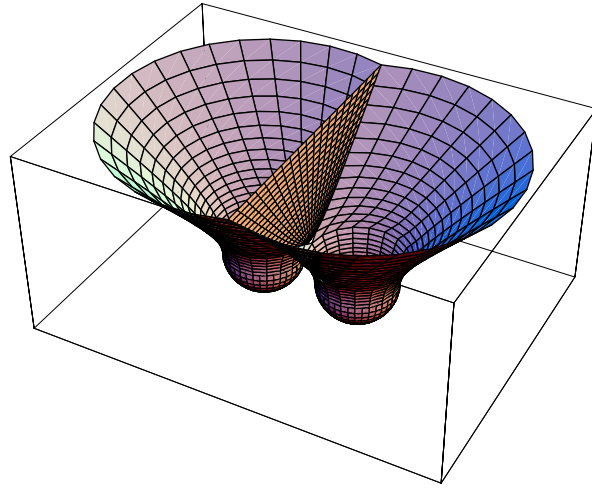
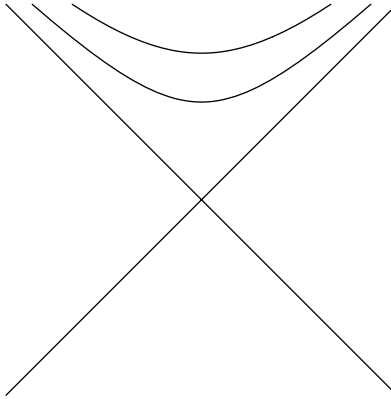


FIG. 3. **Dynamics of vacuum decay through the nucleation of a single bubble.** We show the nucleation through quantum tunneling and the subsequent classical evolution of a single bubble (for simplicity in Minkowski space). As before, time increases in the vertical direction and the horizontal direction indicates one of the four spatial directions. The scalar field is constant on the solid curves. In the lower part of the diagram the nucleation of the bubble is represented by the concentric circles. However, since the inherently quantum mechanical process that produces the initial critical bubble cannot be observed without altering its outcome, it is best to regard this part of the evolution as a quantum mechanical black box whose inner workings cannot be observed. During the subsequent classical expansion of the bubble, the velocity of the bubble wall approaches c .



(a)



(b)

FIG. 4. **The geometry of the collision of two bubbles.** The left panel indicates the collision of two bubbles, represented in the thin wall limit with $(2+1)$ dimensions shown. The right panel indicates a cross section of the plane exactly midway between the two nucleation centers.

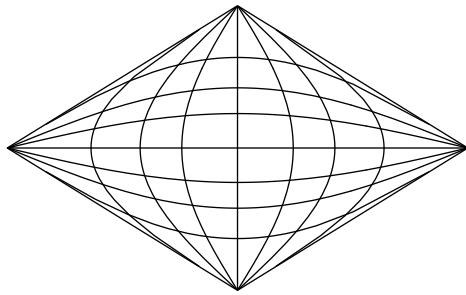
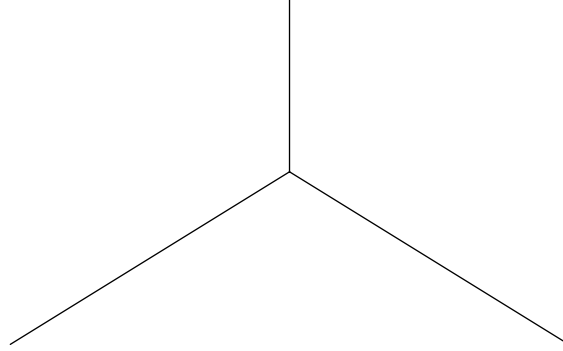
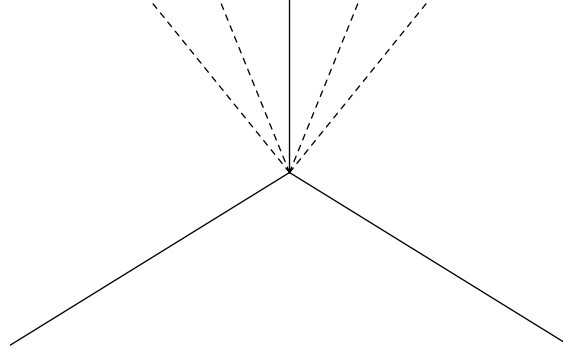


FIG. 5. **Fate of a single AdS bubble.** The bubble interior with the geometry of AdS space is indicated. The scalar field is constant along the surfaces indicated by the solid curves. N is the nucleation center and N' is the point at which the timelike geodesics emanating from N first reconverge. The surfaces on which the scalar field is constant are normal to these geodesics.

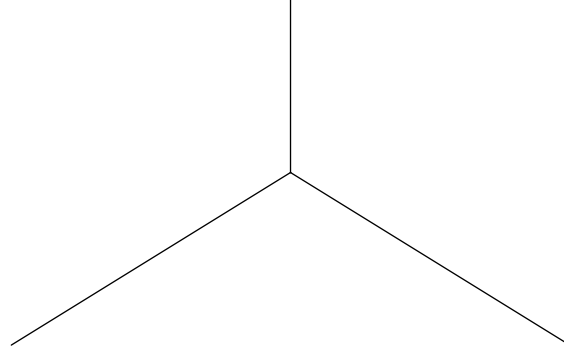


(a)

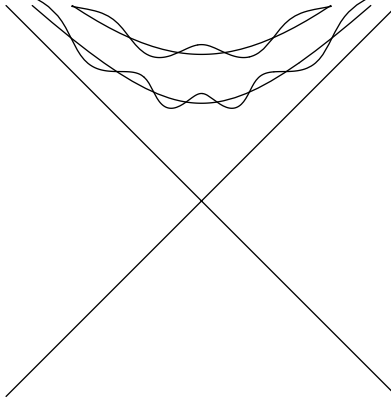


(b)

FIG. 6. **Stress-energy conservation during brane collisions.** Collisions or decays of branes may be represented using as a sort of Feynman diagram in time and the transverse spatial dimension. The three homogeneous spatial directions are suppressed. The vectors $\rho \mathbf{u}$, where ρ is the density on the brane in the brane rest frame and \mathbf{u} is the vector tangent to the brane, must all sum to zero at the vertex. In the left panel, the collision of two branes, with all the energy going into a single brane in the final state, is represented. In the right panel, the case where some of this energy is emitted as dispersive waves (shown by the dashed trajectories) is indicated.



(a)



(b)

FIG. 7. Perturbations in the thin brane approximation. The displacements χ_1 and χ_2 of the two expanding bubble walls are resolved into displacements χ_+ and χ_- of the local brane. The mode χ_- represents a lateral warping of the local brane. The mode χ_+ advances or retards the moment of bubble collision. The right panel illustrates the distortion of the surfaces of constant cosmic temperature of the local brane.